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MEASUREMENTS OF THE RADIATION AND LOSS RESISTANCES OF SMALL HF MULTI-TURN MULTI-RESONANT TUNED LOOPS

M. Harper and M.J. Underhill

University of Surrey

SUMMARY

A novel method ('Wideband-Q method') of finding and measuring the various resistance components of small tuned loop antennas has been published by the authors [1] and this is used for the measurement and assessment of a variety of small HF tuned loops.

The new 'Wideband Q' measurement method relies on bandwidth, and hence Q measurements being made over the two to three octave tuning range of such tuned loop antennas. Because the loop inductance as measured remains constant until the self resonance of the loop is approached the Q values also deliver the total combined loss and radiation resistance as measured at the terminals of the antenna. We find from theory, and confirm by measurement, that the several radiation modes of a loop and the identifiable loss mechanisms in general vary with different powers of the frequency. Where two components vary with the same power of frequency we find that we can separate these by varying the loop or loop conductor sizes, or the placement and orientation of the loop in the environment, and then by applying simple multi-factor analysis to the measurements. The results are found to be highly repeatable and to give error limits of a few percent only.

INTRODUCTION

The purpose of this paper is to compare the radiation resistances and measured "intrinsic" efficiencies of single turn loops with those of multi-turn loops of similar sizes. We also observe that dipole radiation mode is much reduced for the multi turn loop, typically to the point of insignificance, particularly when the loop is well balanced [2].

We define the "intrinsic" efficiency of an antenna as one minus the ratio of the antenna conductor loss resistance of all effective radiation modes. The values are those found at the terminals of the antenna by the novel measurement method. We define the 'environmental' efficiency by including the resistance component from the near-field losses that can be measured at the antenna terminals.

The small tuned (magnetic) loop antenna typically consists of a single turn loop, tuned by a single capacitor, with a subsidiary input loop or gamma match. Over the HF 1.7 to 30 MHz frequency range loops with diameters of 0.8 to 1.2 metre can have radiation efficiency of no worse than 80 to 90% depending on construction, and can operate with powers of a few hundred watts. A loop with a copper tube diameter of greater than 20 to 30mm can be capacitatively tuned over a ten-to-one frequency range with better than 90% efficiency.

The classic formula $3.12 \times 10^4\ [A^2/\lambda^4]$ for the series radiation resistance of a small single turn tuned loop antenna is based on theory that is well founded and provably correct [3]. But measurements show that it predicts radiation resistances which for loops less than $\lambda/160$ in diameter are about a thousand times less than the measured values. We find that this mode only becomes significant at the higher end of the loop tuning range, and more so if the frequency is above about 10MHz.

ANTENNA MODEL AND MEASUREMENT OF Q

Shown in Figure 1 is the model for a tuned loop antenna. By using Laplace Transforms we can find the input impedance of this circuit as:-

$$Z(s) = sL_2 + r_a^{-2}\left[(sL_1 + R_{tot})^{-1} + sC\right]^{-1}$$

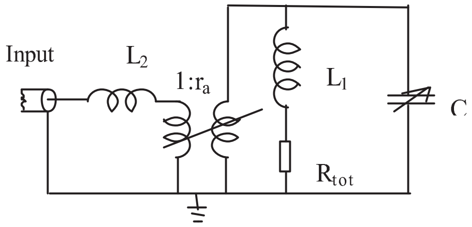


Figure 1 Circuit model of tuned loop antenna

L_1 is the loop inductance and the tuning capacitor C sets the frequency of operation. r_a is the step-up ratio of the equivalent input transformer. The input (match circuit) inductance is L_2.

R_{tot} combines all radiation and loss resistances. In general each of these varies with frequency differently. The equation below represents a novel method for combining all radiation and loss resistances, on the basis that they are lightly coupled to each other. And not connected either fully in series or fully in parallel.

$$R_{tot} = \left(R_{loop}{}^a + R_{trad}{}^a + R_{dip}{}^a + R_{loss}{}^a + R_{env}{}^a + R_{ground}{}^a\right)^{\frac{1}{a}}$$

We find from the measurements that the variable parameter "a" gives the best fit to the results when it is of the order of 2. This means that the resistances are combined in a 'Root Mean Square' operation.

However if ‘a’ is selected to be 1 a 100% coupling is implied, and then there is a large disagreement between the model and the reported measurements. Then we find that it is impossible to choose a set of extracted parameter values that give a reasonable fit over the measured loop tuning range.

We choose and define the model component values and their inter-relationships as:

$$R_{loop} = \frac{\omega \cdot L_1}{Q_{ml}} \qquad (1)$$

$$Q_{ml} = Q_{il} D_{loop}^{-1} \qquad (2)$$

with $Q_{il} = 300$ to 600

$$R_{trad} = \left(\frac{k_t \cdot f \cdot D_{loop}}{3 \cdot 10^8} \right)^4 \cdot 10\pi^6 \qquad (3)$$

with $k_t = 1$ traditionally

$$R_{dip} = 200\left(k_{dip} \cdot f \cdot D_{loop} \cdot \pi / 2\right)^2 \qquad (4)$$

with $k_{dip} = 1$ to 2

$$R_{loss} = 7.07 \cdot 10^6 \cdot \pi \cdot D_{loop} \cdot D_{tube}^{-1} \cdot f_{MHz}^{0.5} \qquad (5)$$

for copper

$$R_{env} = k_e \cdot D_{loop}^2 \cdot f_{MHz}^{0.5} \qquad (6)$$

$k_e = 0.005$ to 0.65

$$R_{ground} = k_c \cdot D_{loop}\left(1 + f^2 \cdot f_g^{-2}\right) \qquad (7)$$

$k_c = 0.02$ to 0.2
$f_g = 2$ to $10MHz$

The use of Eqns. 1 and 2 for Q_{ml} is a novel proposal. Without this, it is not possible for Q_{BW} from the proposed model to be able to match the observed results for Q_{meas}. Note that in this model the Q of the basic loop, Q_{ml}, does not vary with frequency, but it is inversely proportional to loop diameter, D_{loop}. But we find that the intrinsic loop Q factor Q_{il} can be considerably increased by an adjacent low-loss reflector (or in a screened room).

Eqn 3 is the traditional book formula for loop radiation resistance if $k_t = 1$. It is usually very small and negligible unless the loop can be tuned above its self resonant frequency. In practice it is sometimes useful to set this to a higher value, 2 to 3, because this can then represent the effect strong ground reflection is found to have on the loop impedance. For multi turn loops there is an additional factor N^2 to account for the number of turns.

Eqn 4 represents the balanced loop also acting as a folded dipole if $k_{dip} = 1$. If unbalanced and with a ground-plane, k_{dip} can rise typically to 2.

Eqn 5 represents the copper tubing $f^{0.5}$ losses; this is increased by N (number of turns) for multiple turn loops. Eqn 6 represents $f^{0.5}$ losses in the environment from adjacent walls from, for example, reinforced concrete, heat reflecting aluminium wall cladding, or anechoic absorbent material.

Eqn. 7 represents a ground wave coupling effect that is found in practice. It appears as an approximately constant resistance down to a cut-off frequency f_g of typically 3 to 30 MHz. This term has been found to be essential to be able to match the model to the results for loops close to a real ground. It appears that it could become more dominant for larger loops at low frequencies. Further measurements are required to confirm this.

The above parameter values have been suggested on the basis of the measurements that have been made so far. Also for much larger loop sizes and at much lower frequencies the formulas may have to be extended to be valid for all loop sizes with a single set of parameter values. As described the formulas give (very) good agreement over the HF range of 1.5 to 40MHz, for loops with diameters of up to a few metres.

‘Wideband-Q measurement’ method

Accurate and consistent Q measurements over a wide loop tuning range are required so that the several contributions to R_{tot} may be separated and individually measured.

Accurate Q values can be obtained from SWR measurements once the antenna is balanced and matched and if very short cable lengths are used between the source, SWR bridge and antenna. The HP 8457 network analyser can usefully compute out short cable lengths, but only if the cable dielectric and conductor losses are very small. The MFJ 259 portable antenna analyser has the advantage of small size and no connection to mains cables. There is no discernible difference in the results obtained with each of these but the HP 8457 is more convenient to use with balanced antennas where the mains cable has little or no effect.

Unbalance of a loop generates RF currents on the outside of a coaxial feeder (or into the common mode of a balanced feeder). The radiation caused has a substantial effect on the measured Q values as might be expected. Uncontrolled unbalanced feeder resonances reduce the accuracy of our method, giving experimental errors of typically no more than about 10%

A ‘perfect’ 1;1 SWR match at the desired measurement frequency is required for the measurement of the overall loop Q factor, Q_{BW}, by SWR. The twisted wire gamma match can nearly always provide this. It conveniently consists of a length of wire, being a small or large fraction of a half circumference, with a suitable tapping point found for this on the main loop using a ‘crocodile clip’ or similar movable clamp. Wire spacing conveniently provides a means of fine adjustment. Note that it is not unusual for there to be two possible matching points. The choice does not affect the measured Q values provided that the loop is reasonably balanced.

The 3dB bandwidth of a tuned circuit corresponds to a reflection coefficient with a modulus of $\rho = \left|(1-(1\pm j))/(1+1\pm j)\right| = 1/\sqrt{5}$ and this corresponds to an SWR of 2.62. Thus, Q_{meas} is obtained from measurements of the frequencies, $f1$ and $f2$, at which the SWR degrades to 2.62:1. Then $Q_{meas} = (f1 + f2)/2(f1 - f2)$.

MATCHING THE MEASUREMENTS AND THE SIMULATED RESULTS FROM THE MODEL

To fit the measured results to the mathematical model, the following model coefficients have been chosen for six different examples (1 to 6), and the corresponding measured and simulated Q curves are plotted in Figures 2 to 7.

Example	D_{loop}	D_{tube}	k_e	k_t	k_c	f_g	k_{dip}	Q_{il}
1	1m	8mm	0.2	6.3	-	-	5	1000
2	1m	8mm	-	1	0.8	2	-	640
3	1m	8mm	0.01	1	0.48	3.5	-	846
4	1m	8mm	0.4	1	0.5	10	-	390
5	1m	8mm	-	1	0.96	10	-	288
6	1m	22mm	0.049	1	0.05	2	.25	840

Table 1. Parameters for loop antenna model

Errors between measured and simulated Q bandwidth values with specific efficiencies

The error between the measured results and the simulated results from the model has been computed over the loop frequency range and is shown in Table 2. The table also shows efficiencies taken at the lowest and highest frequency points over the loop tuning range.

Example	Average error over frequency range	Efficiency at lowest frequency	Efficiency at highest frequency
1	7%	79%	99%
2	2.3%	80%	81%
3	1.7%	85%	85%
4	2.5%	90%	92%
5	1.4%	92%	93%
6	2.2%	88%	91%

Table 2. Efficiencies and errors of loop model

COMPARISON OF SIMULATED AND MEASURED RESULTS

A three turn loop antenna of 8mm diameter copper tubing of 8mm was constructed with a diameter of 1m. (and with 1 to 2mm turn spacing). Also a 1.2m single turn loop with a copper tube diameter of 22mm was constructed for the purpose of this paper. We now address the results taken by the 'Wideband-Q measurement' method.

Example 1: Three turn unbalanced loop in anechoic chamber

A three turn loop was placed in the anechoic chamber and was configured into an unbalanced mode of operation (feed-point next to one side of tuning capacitor). The measurements were taken using the Q-bandwidth technique over the frequency range 1.5 MHz to 10MHz. Figure 2 shows the practical results (blue) compared to the simulated (red). Good agreement can be seen. The 'fitting' parameters are given in table 1 and efficiencies and errors presented in table 2.

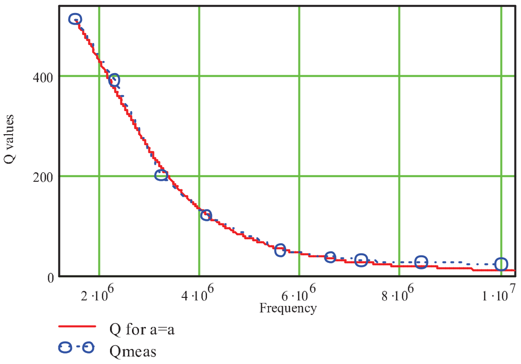


Figure 2 Example 1: Three turn unbalanced loop (1m diameter – 8mm tube) in an anechoic chamber

Example 2 Three turn balanced loop in anechoic chamber

The same 3 turn loop was then configured in a balanced mode and the measurement performed once again over the frequency range 1.5MHz – 4.5MHz (Figure 3) See tables 1 and 2 for matching parameters, and for efficiency and errors.

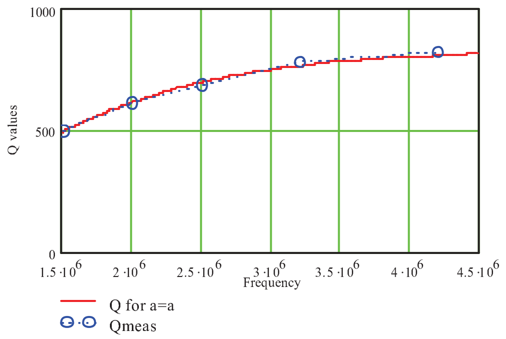


Figure 3 Example 2: Three-turn balanced loop (1m diameter – 8mm tube) in an anechoic chamber

Again good agreement is obtained between the model and the measurements. Note the way in which the overall Q is reduced markedly at the higher frequencies. The efficiency is apparently reduced, because the dipole mode radiation has not been included in this case.

Example 3: Three turn balanced loop in open space

The 3 turn loop was placed in an expansive open space on the University of Surrey campus. The weather at the time is described as sunny and the ground was dry. Table 1 shows the parameters required to 'fit' the model with the results, and Table 2 shows the efficiency and errors for this experiment.

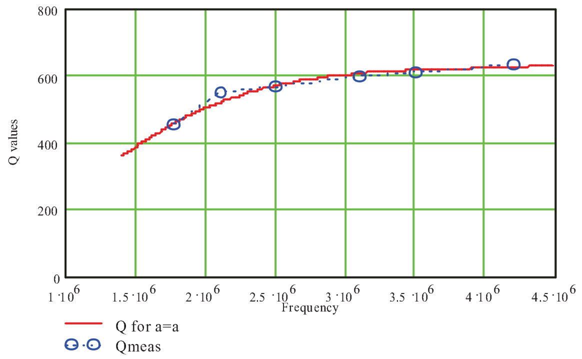


Figure 4 Example 3: Three-turn balanced loop (1m diameter – 8mm tube) outside

Note that the Q is lower because of the radiation that is occurring. On the other hand there are ground losses but these are lower than the wall losses of the anechoic chamber

Example 4: Two turns of three-turn balanced loop in an anechoic chamber

The same 3 turn loop antenna is now configured so that the tuning capacitor resides over 2 turns rather than 3 Fig (4) and the spare turn is shorted out. The measurements are conducted in the anechoic chamber and are taken over the frequency range 3MHz to 8MHz. Table (1-3) and Table (2-3) shows the parameters needed to match the model to the results and the corresponding efficiencies.

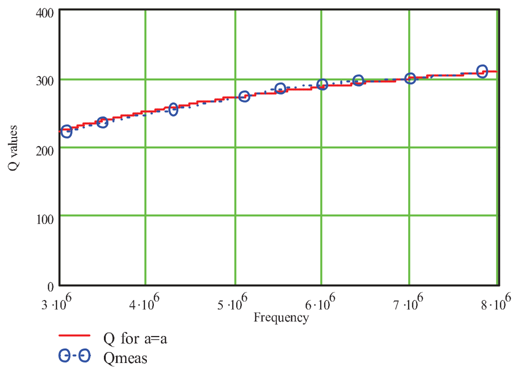


Figure 5 Example 4: Two turns of three-turn balanced loop (1m – 8mm diameter) in an anechoic chamber

We can compare this example with example 1, Note that the Q (at 4MHz) has been reduced to about one third but the estimated radiation efficiency is a bit better at low frequencies and a bit better at high frequencies.

Example 5: Two turns of a three turn balanced loop in an open space

The loop antenna is placed in the same outdoor position as in figure (3) and the results shows in Fig (5) Table (1-4) and Table (2-4) shows efficiencies and the matching parameters for the 3MHz to 8MHz range for this particular antenna.

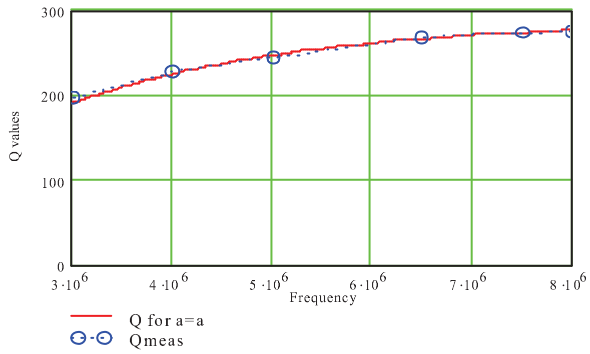


Figure 6 Example 5: Two turns of a three-turn balanced loop (1m diameter – 8mm tube) in an open space

Much the same comments apply as in the previous example.

Example 6: Single turn balanced loop in an anechoic chamber

A single turn loop antenna is placed in the anechoic chamber and measurements are taken over the frequency range of 3MHz to 12MHz see Fig (6) the parameters for the model to fit are given in Table 1and errors and efficiency are given in Table 2.

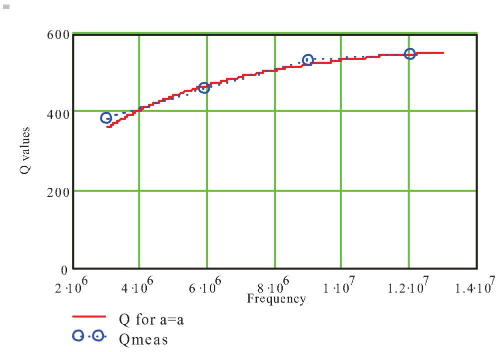


Figure 7 Example 6: Single turn balanced loop (1.2m diameter – 22mm tube) in an anechoic chamber

Note that the Q of this 1.2 m diameter loop is considerably reduced as compared with the 1m loop of example 2, figure 3 also in the same anechoic chamber.

COMPARISON WITH PREVIOUS METHODS OF EFFICIENCY MEASUREMENT

Our novel 'Wideband-Q' method [1] extracts practically all the 'information' that exists in a set of measurements made over a wide band of frequencies to be able to separate the various loss and radiation resistances. We can compare our method with two previous attempts at estimating efficiency from Q measurement [4, 5]

The 'Q factor' method as presented in [4] suffers from three inaccuracies. The first is the incorrect assumption that the 3dB bandwidth point occurs when the reflection coefficient is $|\rho| = 0.5$ and the SWR is 5.83. As shown above the correct values are $|\rho| = 1/\sqrt{5}$ and SWR = 2.62. The losses predicted in the paper are therefore nearly a factor 2 too high.

The second wrong assumption is that the traditional loop radiation mode is the only one that applies and this is can be represented by the traditional radiation resistance formula. The third area of inaccuracy, correctly noted, is that in a (vehicular) environment what constitutes the Chu sphere containing the antenna is uncertain.

Our method is thus a very significant improvement over the Wheeler Cap method. For example we find that the loop resistivity of the Cap wall material is always added to the loop resistivity. This is not as reported by Smith [5]; he only includes very near field losses in his (complicated) analysis. Thus Wheeler Cap measurements are always too low (unless super-conducting walls are used!).

As already mentioned our empirical method of combining several radiation and loss resistances [1] is the only way we could fit the measured results from several different environments to the simple 'lumped' circuit model as shown in Figure 1. From our model the effect of the two methods of combining the resistances on the efficiency measurements are shown in Figure 8 for a 1.6m diameter loop of 20mm copper tube. No environmental losses have been included. The two types of efficiency shown are 'intrinsic' efficiencies which represent RF escaping from the surface of the loop conductor. We also plot the 'simple' (added resistances) efficiency of the traditional loop mode that is traditionally taken to be the dominant mode.

At 2MHz these results show a 437:1 discrepancy in the radiation resistances that we measure and predict, and what the traditional loop formula proposes. The resulting loop efficiencies are 1.45% for the traditional prediction, and 85.5% and 89.6% for our two methods of efficiency calculation.

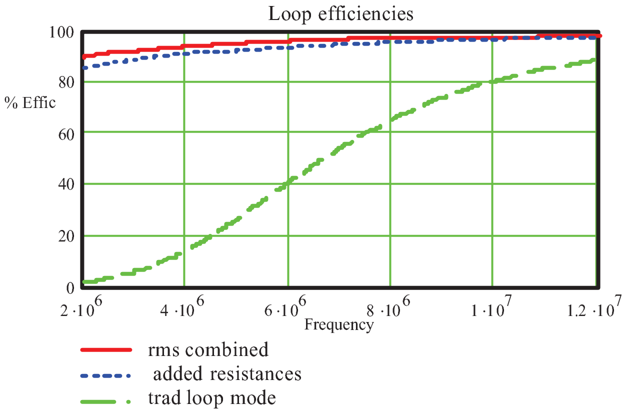


Figure 8. Comparison of estimated efficiencies for 1.6m loop - for (a) rms combined resistances including new loop mode, (b) added resistances including new loop mode, and for (c) added resistances including 'traditional' loop mode only.

BREACH OF THE CHU CRITERION

Small antennas exhibit a large input reactance, a small radiation resistance, a high Q factor and, when tuned to resonance, a small bandwidth of operation. Wheeler [7] first published work on the minimum Q of small antennas in1947. He treated inductors and capacitors just filling a sphere of radius *a*. One year later Chu [8] published a theoretical paper that agreed with the Wheeler more heuristic approach. Chu's approach was to use a partial fraction expansion of the wave impedance of spherical modes that exist outside the smallest circumscribing sphere surrounding the antenna to obtain an equivalent ladder network from which the Q could be found by conventional circuit analysis. The resulting minimum Q criterion is $Q = (ka)^{-3}$.

In his original paper [8], Chu himself did not claim exclusivity and completeness for his proposed small antenna modes. It was others such as Hansen [9] and Balanis [10] who raised the assertion of exclusivity to the level of dogma. Our measurements [1] show an additional and dominant loop mode that firmly contradicts the Chu-Wheeler criterion and gives a well designed loop an intrinsic efficiency above 80 to 90%. The Chu criterion whilst still operative is overshadowed by loop antenna modes that are more dominant. Also we find that the Q does not scale with frequency.

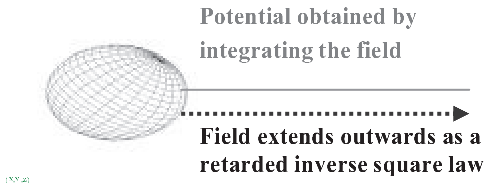
The 'Intrinsic Q factor' is important because it is the Q of the antenna essentially in free space and not effected by the antenna's environment.

Further theoretical work based on a novel line integral potential technique (quasi-scalar) [10] has revealed this to be modified by a weak logarithmic frequency term but the dominant factor still being the antenna size.

ADDITIONAL LOOP MODES FOM A LINE-INTEGRAL (QUASI-SCALAR) POTENTIAL METHOD

A new line integral potential method has been used to predict some of the other modes radiated by the loop shown by our measurements [11].

Essentially the technique uses a Green's function with additional near-field terms which appears to give better accuracy for the near field physics. Like Chu [8] we assume that antenna modes exist surrounding the sphere containing the antenna. A uniform current density is understood to envelop the shell of the sphere creating the field and potential distributions external to the sphere. The field and potential distributions external to the sphere are assumed to be direct continuations of the fields and potentials of corresponding point sources at the centre of the sphere. We assume that the field travels from the source with the velocity of light and obeys the inverse square distance law, and that we can then obtain the potential by integrating the field radially from infinity back to the point of interest on the surface of the sphere.



To find the Q values we first calculate the power radiated at each point of the sphere as the current density times the quadrature phase component of the local potential. (What is 'in-phase' and what is 'quadrature' requires appropriate definition.) The stored energy is the current density times the in-phase component of the local potential and this is derived directly from the measured or calculated surface inductance. The quadrature phase component of the local potential is an induced component from every other point on the sphere and this is well approximated by assuming an equivalent central source for this with a distance time delay. 5 examples of how the Chu criterion has been violated are given in a paper to be published by the authors [11].

CONCLUSIONS

The new Q bandwidth method and model can be used for multi-turn loop antennas.

Our results show that the dominant loop radiation mode does not scale with frequency.

Single turn loops have a comparable efficiency compared to multi-turn loops and also excite a dipole mode of radiation. Multi-turn loops have little or no dipole mode.

It has been found that a single turn loop has a wider continuous tuning frequency range. Multi-turn loops with tapped or shorted turns can have a multiple tuning ranges but these appear to be less efficient.

A practical and theoretical conclusion is that the Chu-Wheeler criterion cannot safely be used as a basis for refuting small antenna measurements.

To be confirmed by further experimental work: 2 turns in parallel will have better efficiency than 2 turns in series and will also have a wider bandwidth.

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